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Building a Quantum Spacetime

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A quantum spacetime is constructed from the free data given at past null infinity. One starts with a field equation for a scalar function Z on the initial surface and then shows that the solution depends on four constants of integration. These constants become the spacetime points and the level surfaces of the scalar function, i.e., Z = const, become null hypersurfaces on the derived spacetime. A phase space together with a complex structure are constructed on past null infinity. This Hilbert space of incoming gravitons possesses a natural foliation which defines superselection sectors on the space of asymptotic quantum states. The dynamics of null surface quantization provides spacetime-valued quantum operators on the superselection sectors. It is shown that the spacetime points themselves become operators with nonvanishing commutation relations.

1. INTRODUCTION

There are several programs attempting to quantize the gravitational field (so far with different degrees of success) [1–4].[2] The canonical quantization program seems to have been successful in constructing a quantum theory of general relativity in a mathematically consistent fashion [6]. The next step in this approach is to give a reasonable physical interpretation. In particular, this program would like to know what possible physical meaning could be given to a "quantum spacetime."

From this perspective, superstring theory is lagging behind since it still is a perturbative theory which needs a background flat metric as the kinematical arena for the quantum operators. However, neither approach is able to provide a mathematical model for a quantum spacetime since both need a differentiable manifold to define quantum fields or strings.

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[2] See refs. 5, e.g., for the problem of the meaning of a quantum spacetime.

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Recently, by applying a quantization procedure to the null surface formulation (NSF) of general relativity [7], we obtained some rather surprising results. We observed that the spacetime points themselves must be "quantized," i.e., turned into operators with commutation relations, etc. A straightforward consequence of this result is that the manifold idea itself must be changed. This is not simply an empty conjecture; there is in principle (in full theory) and in practice (in linear theory) a means to calculate the commutators between individual spacetime points [9]. The only piece of kinematical or background arena left in the NSF construction is the past null boundary of spacetime, denoted by $\mathscr{I}^-$. This initial surface serves as the natural lab where incoming gravitons are produced. Everything else is constructed from the solutions to the NSF field equation.

The outline of this review is as follows. In Section 2 we introduce the main variable of the NSF and give a brief summary of the geometrical constructions needed for this work. In Section 3 we present the field equations, and in Section 4 we outline the construction of the spacetime as well as a canonical coordinate system that allows us to identify points in a natural way. In Section 5 we discuss what happens when we try to turn the new point of view into a quantum theory, and in Section 6 we provide a phase space for these quantum operators. In Section 7 we discuss possible meanings and ramifications of these ideas.

2. SCRI, THE FREE DATA, AND LIGHT CONE CUTS

Consider an asymptotically flat spacetime M with (past) null boundary Scri ($\mathscr{I}^- = S^2 \times \mathbb{R}$). Let $(\zeta, \bar{\zeta})$ be a complex stereographic coordinate on S^2 which labels the generators of $\mathscr{I}^-$, and let $u \in \mathbb{R}$ be an appropriately normalized parameter along the generators. Thus $(u, \zeta, \bar{\zeta})$ are the Bondi coordinates on $\mathscr{I}^-$. The free data for Einstein's equations are then associated with a connection (the Bondi shear) on $\mathscr{I}^-$, and can be specified by the choice of a complex spin-weight-2 field $\sigma(u, \zeta, \bar{\zeta})$ on $\mathscr{I}^-$. The space of all such fields (σ), together with the appropriate symplectic structure, constitutes the reduced phase space of general relativity [10].

Consider now the past light cone from a point x^a in M. The intersection of this light cone with $\mathscr{I}^-$ is called a *light cone cut*. It can be locally described as

$$u = Z(x^a, \zeta, \bar{\zeta}) \tag{1}$$

This function Z, which depends on six variables, contains all the information of the conformal structure, i.e., its knowledge is completely equivalent to knowledge of the metric up to rescaling [11]. It is clear that Z is a nonlocal variable since it is determined from the Riemann curvature along the path of each null geodesic. It is rather remarkable that for pure gravity, i.e., for

Ricci flat metrics, a field equation for Z equivalent to the Einstein vacuum equations can be derived. Details of the field equation are discussed in the next section.

Assuming a solution Z is given, we can introduce the following scalars:

$$u = Z(x^a, \zeta, \bar{\zeta}), \qquad \omega = \eth Z, \qquad \bar{\omega} = \bar{\eth} Z, \qquad R = \eth\bar{\eth} Z, \tag{2}$$

where the $\eth$(eth) and $\bar{\eth}$(eth-bar) operators are essentially partial derivatives with respect to ζ and $\bar{\zeta}$, respectively. For each $(\zeta, \bar{\zeta}) \in S^2$ the scalars (2) define a coordinate system. An alternative notation found in the literature is

$$(u, \omega, \bar{\omega}, R) = (\theta^0, \theta^+, \theta^-, \theta^1) = \theta^i$$

with its gradient and dual vector bases denoted by $\theta^i_{,a}$, and θ^a_i, respectively. This coordinate system is very useful in defining the notion of quantum spacetime points.

3. THE FIELD EQUATIONS FOR *Z*

We review here the null surface formulation of classical general relativity for asymptotically flat spacetimes [8]. By construction, the solutions to the NSF field equations automatically produce null hypersurfaces of a regular, radiative spacetime. Instead of having field equations on a 4-dimensional manifold for the metric of the spacetime, we give equations at $\mathscr{I}^-$ for a scalar field Z. The level surfaces of this function then define the null hypersurfaces of the spacetime.

From the surfaces themselves, by differentiation, a (conformal) metric can be obtained (the surfaces themselves are then automatically characteristic surfaces of this conformal metric). The equations simultaneously determine a choice of conformal factor such that the metric automatically satisfies the vacuum Einstein equations. In other words, the vacuum Einstein equations are formulated as equations for families of surfaces and a single scalar conformal factor. In our present discussion we will be primarily interested in only the characteristic surfaces. Our point of view will be that we are given the equation for the surfaces and we are interested now in information that we can retrieve from knowledge of the solutions to the field equations.

As mentioned in the previous section, the cut function $Z(x^a, \zeta), \bar{\zeta})$ represents the intersection of the past light cone from x^a with $\mathscr{I}^-$. Its specific form depends on the effect of the curvature of the spacetime on the past cone. In particular, if the underlying spacetime represents the self-interaction of incoming gravitational radiation from $\mathscr{I}^-$, then the curvature tensor is completely determined by the data given at $\mathscr{I}^-$. These data are given by a complex-valued spin-weight-2 function $\sigma(u, \zeta, \bar{\zeta})$ which can be given freely. The data restricted to the lightcone cut Z become $\sigma(Z, \zeta, \bar{\zeta})$. A main feature of the NSF is that these free data are used as a source term in the field equation for Z [7, 12]. Since this equation is very complicated, we present here an equation which is equivalent to the Einstein equations at the linearized level. By doing so, we simplify the treatment of the quantization procedure without losing our point of view toward the subject. We start with the following equation at $\mathscr{I}^-$:

$$\eth^2\bar{\eth}^2 Z = \eth^2\bar{\sigma}(Z, \zeta, \bar{\zeta} + \bar{\eth}^2\sigma(Z, \zeta, \bar{\zeta}) \tag{3}$$

Since we are looking for regular solutions to this equation, we can rewrite it in integral form as

$$Z(x^a, \zeta, \bar{\zeta}, [\sigma]) = x^a l_a(\zeta, \bar{\zeta}) + \int_{S^2} d^2\eta \, [G(\zeta, \bar{\zeta}, \eta, \bar{\eta})\sigma(Z, \eta, \bar{\eta}) + c.c.] \tag{4}$$

where $G(\zeta, \bar{\zeta}; \eta, \bar{\eta})$ is a known Green's function [13], the brackets represent functional dependence on the free data, l_a represents the $l = 0, 1$ spherical harmonics, which are annihilated by the $\eth^2\bar{\eth}^2$ operator, and x^a are four constants of integration. Note that although the field equation is given at $\mathscr{I}^-$, the spacetime points enter as parameters in the solution.

The linearized solution to (5) is given by

$$Z(x^a, \zeta, \bar{\zeta}, [\sigma]) = x^a l_a(\zeta, \bar{\zeta}) + \int_{S^2} d^2\eta \, [G(\zeta, \bar{\zeta}; \eta, \bar{\eta})\sigma(x^a l_a(\eta, \bar{\eta}), \eta\bar{\eta}) + c.c.] \tag{5}$$

4. BUILDING A SPACETIME

We now assume to have a solution $Z(x^a, \zeta, \bar{\zeta}, [\sigma])$ and derive several geometrical structures on the solution space x^a. We emphasize again that spacetime points x^a arise as a consequence of the dynamics, i.e., Eq. (5). The only kinematical construction is the null boundary $\mathscr{I}^-$.

- The characteristic surfaces of the spacetime are described as follows: For fixed values of $(u, \zeta, \bar{\zeta})$, the equation

$$Z(x^a, \zeta, \bar{\zeta}; [\sigma]) = u \tag{6}$$

describes a characteristic (or null) surface in terms of the given local chart x^a on our manifold. In fact, the null surface is the future lightcone of the point $(u, \zeta, \bar{\zeta})$ on $\mathscr{I}^-$. As the value of u varies [for fixed $(\zeta, \bar{\zeta})$] we have a one-parameter foliation (of a local region) by the characteristic surfaces. The $(\zeta, \bar{\zeta})$ then label a sphere's worth of these null foliations; equivalently, for each point x^a, as the $(\zeta, \bar{\zeta})$ vary, we obtain a sphere's worth of characteristic surfaces through x^a. An alternate interpretation of the function $u = Z(x^a, \zeta,$

$\bar{\zeta}$; $[\sigma]$) is that, for fixed point x^a, it describes the lightcone cut of x^a; i.e., the intersection of the future lightcone of x^a with $\mathscr{I}^-$.

Assuming that the Z satisfies our differential equations, one can then, in a prescribed fashion, express a conformal Einstein metric in terms of derivatives of Z [7]. In what follows, we assume that the function Z is always implicitly associated with an appropriate conformal factor that guarantees an Einstein metric. Notice, first, that all the conformal information about the spacetime is contained in the knowledge of $Z(x^a, \zeta, \bar{\zeta}; [\sigma])$ and, second, that the (Einstein) conformal factor itself depends on the data. (For the sake of simplicity of presentation, we have slightly simplified the discussion. See ref. 7 for the details.)

• It follows from the above considerations that, for each fixed value of $(\zeta, \bar{\zeta})$, as u varies, the Z describes a foliation by null surfaces. Then the coordinate system given in Section 2 is a null or characteristic coordinate system which functionally depends on the free data. To emphasize this fact we write

$$u = Z(x^a, \zeta, \bar{\zeta}, [\sigma]) \tag{7}$$

$$\omega = \eth Z(x^a, \zeta, \bar{\zeta}, [\sigma]) \tag{8}$$

$$\bar{\omega} = \bar{\eth} Z(x^a, \zeta, \bar{\zeta}, [\sigma]) \tag{9}$$

$$R = \eth\bar{\eth} Z(x^a, \zeta, \bar{\zeta}, [\sigma]) \tag{10}$$

The geometrical meaning of the coordinates $(u, \omega, \bar{\omega}, R)$ is discussed below. Note that we automatically have a sphere's worth of these characteristic coordinate systems (a single coordinate system for every value of $\zeta, \bar{\zeta}$).

With the notation

$$\theta^i = (\theta^0, \theta^+, \theta^-, \theta^1) = (u, \omega, \bar{\omega}, R) \tag{11}$$

we have that

$$\theta^i = \theta^i(x^a, \zeta, \bar{\zeta}; [\sigma]) \tag{12}$$

is a coordinate transformation from the "old" coordinates x^a to a sphere's worth of null coordinate systems θ^i. It should be stressed that the θ^i contain the full information about the solutions of the conformal Einstein equations, through their dependence on the data σ.

• Equation (12) can be algebraically inverted to express the local coordinates x^a in terms of the θ^i and $\zeta, \bar{\zeta}$:

$$x^a = x^a(\theta^i, \zeta, \bar{\zeta}, [\sigma]) \equiv x^a(u, \omega, R, \zeta, \bar{\zeta}; [\sigma]) \tag{13}$$

The inversion shows that, in the chart θ^i, the x^a depend on the data. Note that, like (12), (13) also contains the full information about the solutions of the conformal Einstein equations; i.e., from (13) a metric conformal to an Einstein metric can be obtained. Though this feature is basic, (13) encodes other information which is, at the moment, of more direct interest to us.

• We thus have two related and complementary interpretations (assume we fix the data σ_B) for Z:

1. Lightcone cuts: For a fixed spacetime point x^a, $u = Z(x^a, \zeta, \bar{\zeta})$ yields its lightcone cut on Scri, $\underline{w}$ yields the angle of intersection of the cut with the generators $(\zeta, \bar{\zeta})$ of Scri, and r is the curvature of the cut.
2. Spacetime points: Given a fixed observation point $(u, \zeta, \bar{\zeta})$ at Scri, $x^a(\theta^i, \zeta, \bar{\zeta})$ gives the coordinate which lies at a parameter distance r along the generator $(w, \bar{w})$ of the past lightcone of point $(u, \zeta, \bar{\zeta})$.

• A Ricci flat metric can be constructed explicitly from knowledge of Z. However, our basic variable naturally yields null surfaces rather than fields on a manifold and we are dealing with a surface theory rather than a field theory. What are the quantum analogs of these structures?

5. A QUANTIZATION PROCEDURE

Adopting Ashtekar's asymptotic quantization procedure [10], we can promote the Bondi free data σ_B at $\mathscr{I}^-$ to a quantum operator $\hat{\sigma}(u, \zeta, \bar{\zeta})$ that obeys the following commutation relations:

$$[\hat{\sigma}(u, \zeta, \bar{\zeta}), \hat{\bar{\sigma}}(u', \zeta', \bar{\zeta}')] = i\Delta(u - u')\delta^2(\zeta - \zeta')\hat{1} \tag{14}$$

where the step function $\Delta(u - u')$ reflects the fact that u and u' are null separated. Note that Z also becomes an operator

$$\hat{Z}(x^a, \zeta, \bar{\zeta}) \equiv Z(x^a, \zeta, \bar{\zeta}, [\hat{\sigma}]) = x^a l_a \hat{I} + \int_{S^2} d^2\eta \, [G(\zeta, \bar{\zeta}; \eta, \bar{\eta})\hat{\sigma} + c.c.] \tag{15}$$

and then from (14) one can obtain commutation relations for the $\hat{Z}$; i.e.,

$$[\hat{Z}(x^a, \zeta, \bar{\zeta}), \hat{Z}(x'^a, \zeta, \bar{\zeta}')] = \Delta_f(x^a, x'^a, \zeta, \zeta') \tag{16}$$

where Δ_f is a generalized Feynman propagator [7].

Using the relationship between Z and the metric g_{ab}, we can construct quantum fields $\hat{g}_{ab}$, or $\hat{C}_{abcd}$, on the spacetime, yielding the standard commutation relations. However, as we will see, we can also derive other consequences that cannot be duplicated using any quantum field theory approach.

There is, however, a potential problem, namely, the meaning of quantum surfaces and quantum spacetime points. Below we provide a tentative interpretation of these operators.

5.1. Fuzzy Lightcone Cuts

Following a quantum mechanical interpretation of the trajectory of a particle $y^i(t)$, we regard $\hat{Z}(x^a, \zeta, \bar{\zeta};)$ as a six-parameter family of operators with $(x^a, \zeta, \bar{\zeta};)$ classical parameters.

The eigenvalues of $\hat{Z}$ yield the space of possible values u. Since a generic state of quantum gravity will not be an eigenstate of $\hat{Z}$, we can only associate a probabilistic interpretation for the lightcone cut of a given point. Likewise, the angle of the cut with the generators of Scri and its curvature are fuzzy.

This analogy, however, cannot be pushed too much since we are not quantizing particle motion, but geometric properties of spacetimes. For example, if we are given a physical state ψ of "quantum gravity" and if $|u; (x^a, \zeta, \bar{\zeta})\rangle$ is an eigenstate of $\hat{Z}$ with eigenvalue u, then we do not expect that $|\langle u; (x^a, \zeta, \bar{\zeta})|\psi\rangle|^2$ is a *probability* distribution since it should have similar problems as the relativistic Klein–Gordon field. Rather, we expect that this amplitude will give us the number of gravitons that are peaked around a plane wavefront characterized by the eigenvalue u.

In this preliminary interpretation, it is the points, i.e., values of u, along the generators of $\mathscr{I}^-$ which are "fuzzy." Thus the lightcone cut of the point x^a appears to be fuzzy.

5.2. Fuzzy Spacetime Points

In the dual formulation the spacetime points become operators. Using the inversion equation, we define the operators

$$\hat{x}^a = x^a(\theta^i; \zeta, \bar{\zeta}; [\hat{\sigma}]) \tag{17}$$

keeping the observation points $(\theta^i; \zeta, \bar{\zeta})$ as c-numbers. (These six parameters define the classical observers at Scri.) A very relevant question: What is the analog of a point in the quantum theory?

A candidate could be a common eigenstate $|x^a; (u, \omega, R), \zeta, \bar{\zeta}\rangle$ of the four operators $\hat{x}^a$; this would correspond to well-defined values of all four coordinates, and thus a well-defined "spacetime point." Let us fix a specific null-coordinate system, by fixing $\zeta, \bar{\zeta}$ corresponding to an asymptotic observer.

It can be shown that for the same values of $(\theta^i; \zeta, \bar{\zeta})$,

$$[\hat{x}^a, \hat{x}^b] = 0$$

Thus, it is possible to define a spacetime point eigenstate. However, we do not expect a generic wave function in quantum gravity to be an eigenstate of x^a. Moreover,

$$[\hat{x}^a(\theta^i; \zeta, \bar{\zeta}), \hat{x}^b(\theta^j; \eta, \bar{\eta})] \neq 0$$

This means one cannot define common eigenstates for the entire spacetime, the points of the manifold are fuzzed out, and the only structure free of quantum fluctuations is Scri.

6. THE SPACE OF INCOMING GRAVITONS

A very important issue was left aside in the previous section. The quantum operators there defined obeyed "formal" commutation relations. We still have the task to construct the associated Hilbert and Fock spaces where those operators should act. Following the rules of geometrical quantization, the Hilbert space will be a properly defined phase space together with a Hermitian inner product

A sketch of this construction follows [14]:

- We start with the solution space $\Sigma = \{\sigma_B\}$ at Scri.
- Σ admits a natural foliation where each leaf is labeled by the value of the mass aspect ψ_2 at i_o.
- On each leaf we introduce a symplectic form Ω,

$$\Omega(\delta\sigma_1, \delta\sigma_2) = \int_{\mathcal{I}} d^3I\,[\delta\sigma_1\delta\dot{\bar{\sigma}}_2 - \delta\dot{\sigma}_1\delta\bar{\sigma}_2] + c.c. \tag{18}$$

where $\delta\sigma_1$, $\delta\sigma_2$ are tangent vectors on $T(\Sigma)$.

- A complex structure

$$\mathbf{J}\delta\sigma = i(\delta\sigma_{pos} - \delta\sigma_{neg}) \tag{19}$$

is then introduced on the phase space. (Note that this operator satisfies $\mathbf{J}^2 = -1$.) Furthermore, we can also introduce a Hermitian inner product,

$$g(\delta\sigma_1, \delta\sigma_2) = \Omega(\sigma_1, \mathbf{J}s_2) + i\Omega(s_1, s_2) \tag{20}$$

- To construct the Hilbert space we need a prescription to select the configuration space, i.e., the space of q's or p's. Since each leaf is not a cotangent bundle nor a vector space, we cannot apply the standard quantization procedure to construct the Hilbert space. Fortunately, there is available a method called geometric quantization which can be applied to each leaf since **J** induces a Kahler polarization.
- It is worth mentioning that there is no relation between Hilbert spaces of different leaves. This suggests the possibility of superselection sectors in

a quantum theory of gravity. (A similar result was obtained when quantizing a spherically symmetric spacetime [15].)

7. SUMMARY AND DISCUSSION

We briefly summarize the steps taken.

- We start with the solution space for radiative spacetimes $\Sigma = \{\sigma_B\}$ and construct a physical phase space (Σ, Ω, J).
- Via the NSF field equations, the Einstein equations are encoded in Z, a functional on σ_B. Z determines a conformal metric and a coordinate system θ^i.
- The θ^i can be inverted to yield the local coordinates x^a of the spacetime points (they become six-parameter functionals on the phase space).
- Following Ashtekar's quantization procedure, we then promote the free data to quantum operators. This is then used to construct the quantum $\hat{\theta}^i$ and $\hat{x}^a$.
- We derive an immediate consequence: A physical state of quantum gravity cannot be an eigenstate of $\hat{x}^a$, the spacetime manifold is not a well-defined entity in quantum gravity.
- We also show that a Hilbert space of incoming gravitons can be constructed on each leaf, but not on the entire Σ. This defines superselection sectors in quantum gravity.

We close with some comments.

- We face a conceptual problem in the full theory: how to construct the operators $\hat{\theta}^i$ and $\hat{x}^a$ in the full theory. There are several technical problems that must be solved before a consistent theory can be formulated. Among them we can mention the ordering problem, the appearance of infinities in a perturbation procedure, or, alternatively, a formulation of a nonperturbative theory.
- In principle our phase space should be equivalent to the reduced phase space of canonical gravity. We have been unable to show this correspondence.
- In our formulation the quantities $x^a(u, \omega, \overline{\omega}, r, \zeta, \overline{\zeta}, [\sigma])$, considered as families of functions on the reduced phase space, are examples of evolving constants of motion. It has been argued that these constants describe evolution in a diffeomorphism-invariant way [16]. Thus, they must be promoted to quantum operators. However, no such quantity was known in general relativity. Here we give an explicit example and construct their quantum analog $\hat{x}^a$.
- We emphasize that ours is not a field theory on a background manifold, rather a theory of surfaces. Our only background structure is Scri equipped with free functions σ. In this sense our picture is a radical departure from quantum field theory: quantum spacetime is ill defined.

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